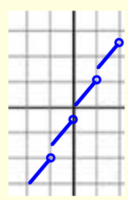
Warm Up # 2-3

Find and describe any discontinuities.

$$f(x) = \frac{x^4 + 7x^3 + 11x^2 - 7x - 12}{(x - 1)(x^2 + 5x + 6)}$$

HW Questions: p. 75

6) Find the points of discontinuity.



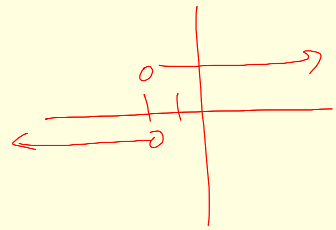
#7-24 Discontinuities? Removable?

11) $f(x) = \dfrac{x}{x^2 + 1}$

12) $f(x) = \dfrac{x - 3}{x^2 - 9}$

16) $f(x) = \begin{cases} -2x+3, & x < 1 \\ x^2, & x \geq 1 \end{cases}$

19) $f(x) = \dfrac{|x+2|}{x+2}$



20) $f(x) = \dfrac{|x-3|}{x-3}$

39) Prove the given function has a zero on the closed interval.

$f(x) = x^2 - 4x + 3$ on $[2, 4]$

$f(x)$ is continuous on $[2, 4]$

$f(2)$	$f(c)$	$f(4)$
-1	0	3

43) Verify the applicability of the Intermediate Value Theorem, then find c.

$f(x) = x^2 + x - 1$ on $[0,5]$; $f(c) = 11$

f is continuous on $[0,5]$ $\quad 11 = c^2 + c - 1$

$f(0) < f(c) < f(5)$

$-1 < 11 < 29$

p. 82 Find the limit.

23) $\lim\limits_{x \to 2^+} \dfrac{x-3}{x-2}$

Given: $f(x) = \frac{1}{(x-4)^2}$ $\quad g(x) = x^2 - 5x$

Find the limit:

35) $\lim_{x\to 4}[f(x)+g(x)]$

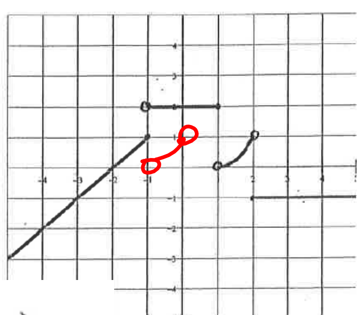
$\infty + (-4)$

∞

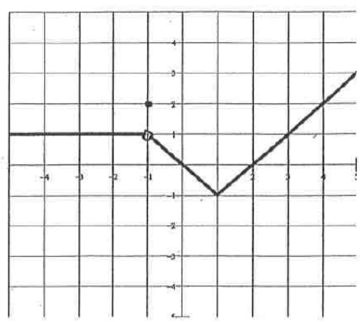
36) $\lim_{x\to 4}[f(x)g(x)]$

38) $\lim_{x\to 4}\frac{g(x)}{f(x)}$

Classwork:



Graph of f



Graph of g

15. $\lim_{x\to 0}(f(x)-10)$

16. $\lim_{x\to 0^-} f(x+2)$

17. $\lim_{x\to 0}(f(x-2)+4)$

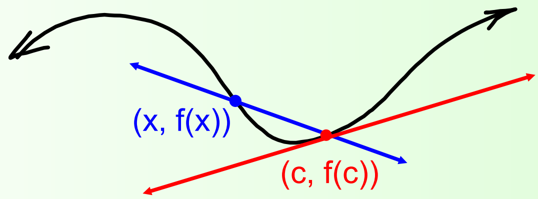
Back to the tangent line problem...

$$m = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

Point - Slope Form
$y - f(c) = m(x - c)$

Move x closer and closer to c

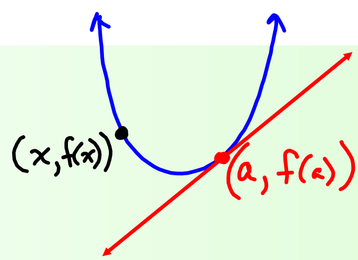
by taking the limit.



You try:

1. $f(x) = x^2 + 3x + 4$
   a) find the slope of the line tangent to f(x) at any point (a, f(a)).

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$



$$\lim_{x \to a} \frac{x^2 + 3x + 4 - (a^2 + 3a + 4)}{x - a}$$

$$\lim_{x \to a} \frac{x^2 - a^2 + 3x - 3a}{x - a}$$

factoring top:
$\underbrace{(x+a)(x-a)} + \underbrace{3(x-a)}$
$(x-a)(x+a+3)$

$$\lim_{x \to a} \frac{\cancel{(x-a)}(x+a+3)}{\cancel{x-a}}$$

$m = a + a + 3$
$m = 2a + 3$ at $(a, f(a))$

2. The line tangent to f(x) has a slope of 3 at the point (1, 2). Find h.

$f(x) = x^2 + hx + t$

2. The line tangent to f(x) has a slope of 3 at the point (1, 2). Find h. $f(x) = x^2 + hx + t$

$$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = 3$$

and $f(1) = 2$

$1^2 + h(1) + t = 2$

$t = 1 - h$

sub in for t

2. The line tangent to f(x) has a slope of 3 at the point (1, 2). Find h. $f(x) = x^2 + hx + t$

$$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = 3$$

and $f(1) = 2$

$1^2 + h(1) + t = 2$

$t = 1 - h$

sub in for t

$$\lim_{x \to 1} \frac{x^2 + hx + 1 - h - 2}{x - 1} = 3$$

Rearrange the numerator & factor.

2. The line tangent to f(x) has a slope of 3 at the point (1, 2). Find h. $f(x) = x^2 + hx + t$

$$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = 3$$

and $f(1) = 2$

$1^2 + h(1) + t = 2$

$t = 1 - h$

sub in for t

$$\lim_{x \to 1} \frac{x^2 + hx + 1 - h - 2}{x - 1} = 3$$

$$\lim_{x \to 1} \frac{x^2 - 1 + hx - h}{x - 1} = 3$$

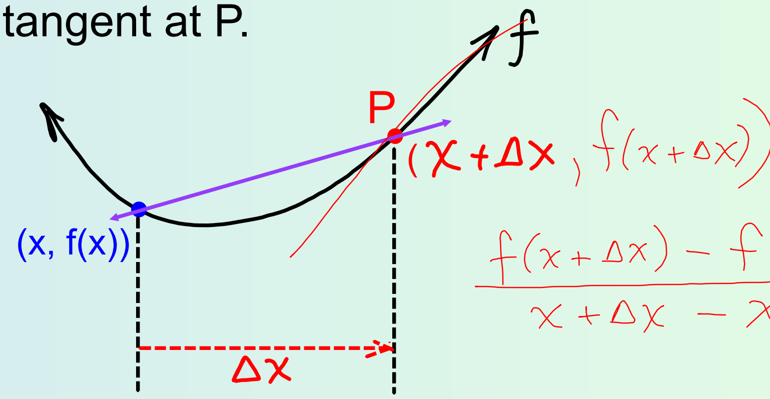
$$\lim_{x \to 1} \frac{(x-1)(x+1) + h(x-1)}{x - 1} = 3$$

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x+1+h)}{\cancel{x-1}} = 3$$

$1 + 1 + h = 3$

$\boxed{h = 1}$

Write a limit formula to find the slope of the tangent at P.



$$\frac{f(x+\Delta x)-f(x)}{x+\Delta x - x}$$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$$

THE DERIVATIVE OF A FUNCTION:

We are now ready to talk about one of two fundamental operations of calculus. The limit used to define the slope of a tangent line is also used to define differentiation.

The derivative of f at x is given by

$$f'(x)=\lim_{\Delta x \to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

provided the limit exists.

$\Delta x \to 0$

minimize the change in x until the slope of the secant line is = the slope of the tangent.

The process of finding the derivative of a function is called differentiation. Some common notations:

$f'(x)$ - read "f prime of x"

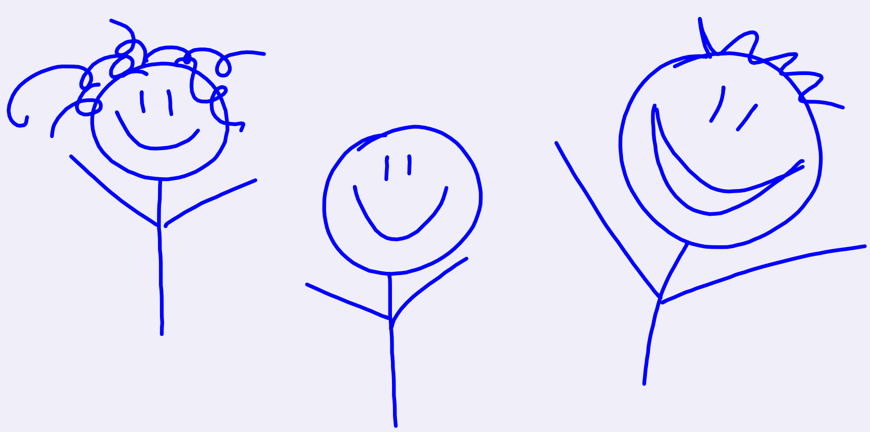
$\frac{dy}{dx}$ - read "the derivative of y with respect to x"

y' - read "y prime"

$\frac{d}{dx}\left[f(x)\right]$ - read "the derivative of $f(x)$ in terms of x"

The Limit Definition of the Derivative:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$



$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \boxed{\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}} = f'(x)$$

Find the derivative of $f(x) = x^3 + 2x$, then find the slope when x= 4.

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \boxed{\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}} = f'(x)$$

Find the derivative of $f(x) = x^3 + 2x$, then find the slope when x= 4.

$$\lim_{\Delta x \to 0} \frac{(x+\Delta x)^3 + 2(x+\Delta x) - (x^3 + 2x)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{1}{\Delta x}\left(\cancel{x^3} + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 \cancel{+2x} + 2\Delta x \cancel{- x^3} \cancel{- 2x}\right)$$

$$= \lim_{\Delta x \to 0} \frac{1}{\cancel{\Delta x}} \cdot \cancel{\Delta x}(3x^2 + 3x\Delta x + (\Delta x)^2 + 2)$$

$$= 3x^2 + 3x(0) + 0^2 + 2$$

$$= 3x^2 + 2 \leftarrow \text{slope}$$

when $x = 4$

$m = 3(4)^2 + 2$

$\boxed{m = 50}$

HW: bottom of page 92

Chapter 2 Review

1 - 39 odds

Friday: Test Ch. 2